

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Algebra

Position-to-term rules for arithmetic sequences

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

13

B1

[1]

Q2.

$3n - 1$

oe eg $2 + (3n - 3)$

B1 $3n + c$ where c can be any value

B2

Additional Guidance

Ignore LHS of formula given eg $T_n = 3n - 1$

B2

Condone $n = 3n - 1$ or n th term $= 3n - 1$

B2

Allow a multiplication sign eg $3 \times n - 1$ or $n \times 3 - 1$

B2

Allow other variables eg $3x - 1$

B2

$3n + - 1$

B1

$3x$

B1

$n3 \dots$

B1

$n3 - 1$

B1

$3nth - 1$

B1

$3nth$

B0

$n3 - 1n$

B0

[2]

Q3.

$4n + 3$

B1

[1]

Q4.

$3n + 4$ or $4 + 3n$

oe eg $7 + (3n - 3)$

B1 $3n (+ \dots)$ or $3n (- \dots)$

B2

Additional Guidance

Ignore LHS of formula given eg $T_n = 3n + 4$

B2

Condone $n = 3n + 4$ or n th term $= 3n + 4$

B2

Allow a multiplication sign eg $3 \times n + 4$ or $n \times 3 + 4$

B2

Allow other variables eg $3x + 4$

B2

$3x$

B1

$n3 \dots$

B1

$n3 + 4$

B1

$3n$ th $+ 4$

B1

$3n$ th

B0

$3n + 4n$

B0

[2]

Q5.

$10n + 1$ or $1 + 10n$

B1 $10n (\dots)$

B2

Additional Guidance

Ignore LHS of formula given e.g. $T_n = 10n + 1$

B2

Condone $n = 10n + 1$ or n th term $= 10n + 1$

B2

Allow other variables e.g. $10x + 1$

B2

Allow a multiplication sign e.g. $10 \times n + 1$ or $n \times 10 + 1$

B2

$n10 \dots$

B1

$n10 + 1$

B1

$n10 + 1n$

B0

Choice e.g. $10n + 1$ and $1n + 10$

B0

[2]

Q6.

$$2n - 12$$

B1

[1]

Q7.

15, 11, 7, 3 as the first four terms

or

$$19 - 4 \times 5 \text{ or } 19 - 20$$

or

-1 as the first negative term

or

4.75

oe

M1

5

A1

Additional Guidance

5n on answer line with 5 in working

M1A0

n = -1 without correct working for M1

M0

4.75n on answer line with no correct M1 values

M0

19 - 4n < 0 with no correct M1 values

M0

[2]

Q8.

55 and 91

B2 for (7), 19, 31, 43, 55, 67, 79, 91

or 55 identified with 0 or 1 incorrect answer

or 91 identified with 0 or 1 incorrect answer

or 55 and 91 identified with 1 incorrect answer

B1 at least 2 correct two-digit numbers from the sequence seen

B3

Additional Guidance

The correct sequence is (7), 19, 31, 43, 55, 67, 79, 91

Ignore continuation of sequence beyond 91

Ignore further working unless contradictory

55 and 91 identified and 5th and 8th terms stated (ignore fw)

B3

55 and 91 identified and answer 2 (or there are 2) (ignore fw)

B3

55 identified and 5th stated (ignore fw)

B2

Condone 5 or 5th as final answer provided there is a clear link to 55 eg

$$12 \times 5 = 60 - 5 = 55 \quad 55 \div 11 = 5 \quad 5 \text{ on answer line}$$

B2

Condone 8 or 8th as final answer provided there is a clear link to 91 eg

$$12 \times 8 = 96 - 5 = 91 \quad 8 \text{ on answer line}$$

B2

[3]

Q9.

13 20 27 and Add 7

or 15 27 39 and Add 12

or 20 15 10 and Subtract 5

or 27 20 13 and Subtract 7

or 39 27 15 and Subtract 12

oe rule

B1 one correct arithmetic progression (using numbers from the list) with no or incorrect rule ie

13 20 27

or 15 27 39

or 20 15 10

or 27 20 13

or 39 27 15

B2

Additional Guidance

Accept the expression for the n th term as the rule

13 20 27 and $7n + 6$ or e.g. $\times 7 + 6$

or 15 27 39 and $12n + 3$

or 20 15 10 and $25 - 5n$

or 27 20 13 and $34 - 7n$

or 39 27 15 and $51 - 12n$

B2

Ignore incorrect expression for the n th term alongside a correct rule

eg 13 20 27 and Add 7 so $n + 7$

B2

13 20 27 and +7 or 7 more or going up in 7s

B2

20 15 10 and five times table (scores for the arithmetic progression)

B1

13 20 27 and $n + 7$ (scores for the arithmetic progression)

B1

Using number(s) not on the list

B0

10 15 20 and Add 5

B0

[2]

Q10.

$98 - 8n$

B1

[1]

Q11.

$7n - 1$ or $-1 + 7n$

oe does not need to be simplified

B1

Additional Guidance

$n7 - 1$

B0

[1]

Q12.**Alternative method 1** Works out n th term of new sequence

Common difference of 5 identified

implied by $5n \dots$ **M1**

$5n + 3$

oe eg $8 + 5(n - 1)$ **A1**

their $(5n + 3) - (n + 1)$

*oe**their $(5n + 3)$ must be a linear expression**condone missing brackets***M1**

$4n + 2$

oe eg $6 + 4(n - 1)$ *ft their $5n + 3$ which must be a linear expression**missing brackets must be recovered***A1ft****Alternative method 2** Works out terms of sequence A and sequence B

2, 3, 4

*sequence A***M1**

6, 10, 14

*sequence B***A1**

Common difference of 4 identified

*ft their 6, 10, 14 which must be a linear sequence for B***M1**

$4n + 2$

oe eg $6 + 4(n - 1)$ *ft their 6, 10, 14 which must be a linear sequence for B***A1ft****Additional Guidance**

Choose the scheme that favours the student

[4]**Q13.****Alternative method 1**

$21 - 17$ or $17 - 21$

or $17 + 4$ or $21 - 4$

or (difference is) 4

or (7th term =) $21 + 4$ or 25

or (4th term =) $17 - 4$ or 13

may be seen as $17 \quad 21$ *4**allow (difference is) -4* **M1**

$17 + (100 - 5) \times 4$

or $17 + 95 \times 4$

or $17 + 380$

*must be using 4**oe calculation that would evaluate to 397*

$5\text{th term} + 95 \times 4$

$$6th\ term + 94 \times 4$$

$$1st\ term + 99 \times 4$$

$$0th \text{ term} + 100 \times 4$$

397

M1dep

A1

Alternative method 2

 $4n$

oe eg $n \times 4$

M1

$$4n - 3$$

oe

A1

397

A1

Additional Guidance

Term to term rule described eg Add on 4 each time

M1

$$a + 5d = 21, a + 4d = 17 \text{ only}$$

MO

Difference shown as 4 then eg $n + 4$

M1

Only eg $n + 4$ or $3n + 4$

MO

$4n - 3$ seen even if not subsequently used

M1A1

$4n$ seen eg $4n + 13$ even if not subsequently used

M1

Correct list going up in 4s stopping at 397

M1M1A1

List going up in 4s with an error or not reaching 397

M1M0A0

No subtraction seen and incorrect difference eg $17 \begin{array}{r} 21 \\ +3 \end{array}$

MO

Alt 2 allow n_4

M1

$$4n - 3 = 100$$

M1A1A0

Allow M1 even if not subsequently used

[3]

Q14.

Alternative method 1

(5th term =) $a + 10b + 4b + 4b$

or (5th term =) $a + 18b$

oe

M1

$a + 6b = 8$

and $a + 18b = 44$

oe correct simultaneous equations

eg

$3a + 18b = 24$ and $a + 18b = 44$

implied by $12b = 36$ or $2a = -20$

M1dep

$b = 3$

or $a = -10$

A1

$a = -10$ and $b = 3$

Alternative method 2

$(d =) \frac{44 - 8}{3}$ or $(d =) \frac{36}{3}$

or $(d =) 12$

any letter

M1

$4b = 12$

oe

M1dep

$b = 3$

A1

$a = -10$ and $b = 3$

A1

Additional Guidance

Correct substitution without writing simultaneous equations scores the first two marks on alt 1

eg

$(a = 8 - 6b \text{ and}) 8 - 6b + 18b = 44$

M1M1

[4]

Q15.

$$750 - 400$$

$$\text{or } 1100 - 750$$

$$\text{or } (1100 - 400) \div 2$$

$$\text{or } 350$$

oe

M1

$$1100 + \text{their } 350 \text{ or } 1450$$

or

$$1100 + 2 \times \text{their } 350 \text{ or } 1800$$

oe

M1dep

Addition or correct total of their Level 1 to Level 5 scores

their Level 4 and Level 5 scores must not be zero or blank
5500 implies M3

M1

$$4250$$

SC2 550

A1

Additional Guidance

SC2 550 is for using Level 5 score 1800 as the highest possible score

Embedded answer

eg $4250 + 1250 = 5500$ without 4250 as their answer

M1M1M1A0

[4]